

### Hayek's Triangle, General Equilibrium, and Keynesian Folly: a first step in bridging the gap between the Spanish-Austrian school and mathematical formalism

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## Hayek's Triangle, General Equilibrium, and Keynesian Folly

A first step in bridging the gap between the Spanish-Austrian School and mathematical formalism

During the 1930s an important debate between two economists took place: one who would become the head of the Bank of England; and one who would receive the Nobel Prize in economics. The winner of the debate was John Maynard Keynes, the eventual bank director. The loser was Friedrich August Hayek, the eventual prize-winner<sup>1</sup>. Keynes was hardly Hayek's intellectual superior and appears to have won because he catered to the political climate of the day -- big government, organized labor, central banking, and even war. After nearly a century of booms, busts, wars, rising unemployment, and an ever-growing income gap there is an important need to revisit the debate and especially the arguments, not of the scholarly banker and political statesman, but of the devoted intellectual and scholar.

Although this model builds on the work of Roger W. Garrison, it is quantifiably more rigorous and perhaps more useful as a result. Like Garrison's general equilibrium model it shows the overwhelming weakness of the Keynesian macroeconomic approach, as well as the need for the re-introduction of sound money as a practical policy solution to today's relentlessly reoccurring global financial and economic crises.<sup>2</sup> Finally, if this paper makes a useful intellectual contribution to the field of economics, then it is its derivation of Hayek's triangle from a perhaps estimable mathematical function and its strengthening of Roger Garrison's effort to bridge the theoretical gap between mainstream mathematical formalism and the Austrian business cycle developed by Ludwig von Mises, Friedrich August Hayek, and others.<sup>3,4</sup>

The model is divided into three components including a profit-maximization problem with a constraining value-added cost function, a utility maximization problem with a national budget constraint, and a private sector investment market whose demand and supply functions are respectively derived from the solutions to the aforementioned optimization problems. In particular the value-added cost function focuses on the

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1 The Royal Swedish Academy of Sciences awarded the 1974 Prize for Economic Science in memory of Alfred Nobel to Professor Gunnar Myrdal and Professor Friedrich Hayek for their "pioneering work in the theory of money and economic fluctuations and for their penetrating analysis of the interdependence of economic, social and institutional phenomena". <[http://www.nobelprize.org/nobel\\_prizes/economics/laureates/1974/press.html](http://www.nobelprize.org/nobel_prizes/economics/laureates/1974/press.html)>

2 Stegemann. (2012, 2013).

3 If the model is of interest to the student of Islamic economics, then it is because it is built on a real economy that views money, not as a speculative financial instrument created as an artifice by private and central bankers for the purpose of wealth redistribution, but as a tradable commodity whose primary purpose is to effectuate the trade of, and investment in real goods and services.

4 This paper is largely inspired by the work of Roger W. Garrison (2001), Jesús Huerta de Soto (1998), Friedrich August Hayek (1976), Jörg Guido Hülsmann (2008), and last, but certainly not least, Ludwig von Mises (1912) that I read in the original only after the fourth draft of this paper.

## The Model's Components

term structure of capital as the key to sustained, long-term, economic growth in the absence of environmental concerns.<sup>5</sup> The utility maximization problem highlights the trade-off between current and future real consumption and the effect of uninvested savings on each. The investment market characterizes the price mechanism by which the aforementioned trade-off between current and future consumption takes place. After a thorough discussion of each component all three components are combined to form the general equilibrium model whose presentation is the ultimate goal of this paper.

## The Model's Components

### Component One: Hayek's Triangle Revisited

#### THE VALUE-ADDED COST FUNCTION

An important short-coming of mainstream economic thought is its failure to reconcile short-term Keynesian economics with long-term growth models. This important short-coming is easily overcome with the introduction of the Austrian business cycle, a cycle most easily understood as a contrast between two economies: a real economy based on the production of real goods and services whose medium of exchange is real money, and a real economy whose medium of exchange consists of money that has been lent into existence by private banks (*ex nihilo* currency or legal counterfeit). An important goal of this paper is to introduce the former so that the latter can be understood more clearly.

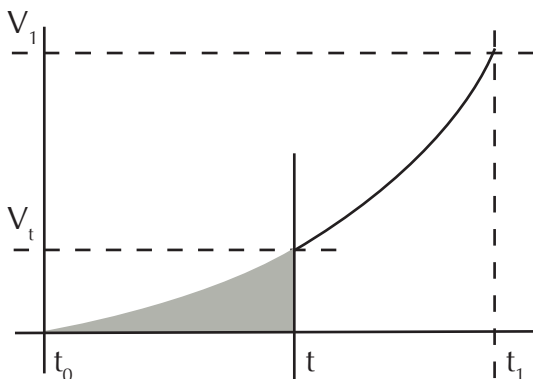


Figure 1. Value-Added Cost Function

The first component of our model begins with a simple triangle derived from the following value-added cost constraint.

$$V_t = V(t, I_0) \quad (\text{equation 1})$$

such that

$$\delta V / \delta t > 0 \quad (\text{equation 2a})$$

$$\delta V / \delta I_0 > 0 \quad (\text{equation 2b})$$

where  $V_t$  = value-added cost between time  $t_0$  and  $t$   
 $t$  = time passed to current stage of production  
 $t_1 - t_0$  = one production cycle  
 $I_0$  = investment at  $t_0$

#### FACTORS OF PRODUCTION

As real investment is merely a transfer of real goods and foregone services from investors to the owners of factor inputs across time, only investment and time need be included as a source of cost to the entrepreneur -- the market agent who facilitates the transfer. In order to understand this more clearly we may think

<sup>5</sup> A well functioning economy can be thought of as a fully nourished carburetor with air and gas in the right proportions. When the gas tank runs dry, however, the carburetor and engine stop. Today's economy is completely dependent on fossil fuels for its existence, and fossil fuels are an exhaustible resource that cannot be easily replaced. There are other environmental concerns, as well. Unfortunately, there are few economic models that are not strictly devoted to the study of environmental economics that take the environment into account; this model is not an exception in this regard.

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of the entrepreneur as both a fund-raiser and producer. As a fund-raiser he buys in the investment market; as a producer he sells in the product market. The entrepreneur succeeds in these two basic transactions only insofar as he can produce and deliver more or higher quality real goods and services than he pays out to the owners of factor inputs who make this allocation, production, and delivery possible. Among the factor owners are, of course, the investors whose plant, machinery, and equipment the entrepreneur employs to enhance the inputs of other factor owners -- namely, those who provide the human effort (labor) and natural resources (land) necessary to achieve the higher level of production. As the focus of our model is the transfer of real goods and foregone services across time, we only model the price mechanism for this transfer. This said, we must account for all forms of factor payment, else our model fails in its most important task.

### FACTOR PAYMENTS

When the entrepreneur obtains funds in the investment market he makes a promise to repay not only what he has borrowed, but also some additional amount that reflects the additional value that he promises to create. This additional amount, as well as the principal, cannot be paid until it is realized in the form of finished real goods and services at the end of the production cycle.<sup>6</sup> These finished real goods and services are then purchased, not only by the investors to whom the entrepreneurs have kept their promise, but also by the owners of other factor inputs currently utilized in other production cycles that proceed concurrently with the just-completed production cycle. The sale and purchase of these goods and services are concomitant insofar as the revenue to the entrepreneur becomes the purchasing power of the factor owners and investors. Although these latter receive payment simultaneously, their payment differs in the time between sacrifice and remuneration. Whereas investors receive payment for their sacrifice made at the beginning of the production cycle, the owners of human effort and natural resources receive payment for their sacrifice in the moment that it is made.<sup>7</sup> To illustrate this point consider figure 2.

The three production cycles labeled  $V'$ ,  $V''$ , and  $V'''$  represent production cycles that run concurrently with the production cycle of length  $(t_1 - t_0)$ . The vertical line that passes through  $t_1$  and the corresponding horizontal lines passing through the points  $V'_t$ ,  $V''_t$ , and  $V'''_t$  represent the stage of production at which each concurrent cycle has arrived within its own respective time-frame. Only the unmarked cycle whose value-added cost is evaluated at  $(t_1, V_1)$  has been completed. It is the finished output from the completion of this cycle that production in all of the other cycles is sustained in its moment of

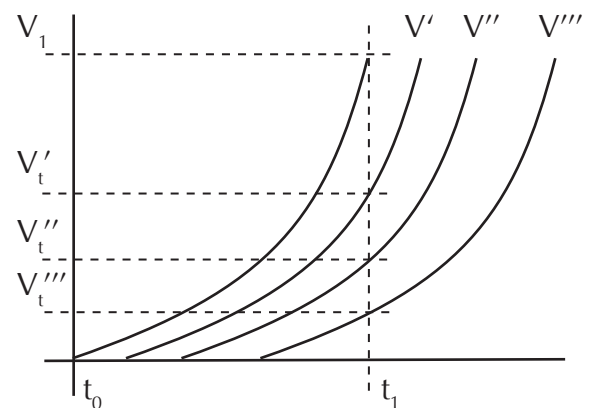


Figure 2. Concomitant Production Cycles

<sup>6</sup> In the case of the sale and purchase of stock the principal is not repaid, but the ownership is retained.

<sup>7</sup> It is important in this context to differentiate between new and old investment. Once an investment project is completed, it generates a net cash flow that is both immediate and continuous. In contrast, these cash flows are non-existent until the project for which the investment was made has been completed. Here, of course, we are referring to new investment above that of mere replacement and operating capital. This point will be further elaborated.

### The Model's Components

completion. It should be clear from this diagram that there exists a continuous flow of finished real goods and services across time. In fact, under stable, zero-growth conditions the final value-added cost  $V_1$  is the same at the end of all production cycles. In effect, one could draw a straight line intersecting each cycle at its top most point. More practically envisioned, each function is likely to differ in both height and length.

During the middle of each production cycle two kinds of transaction take place at each and every  $t$ : viewed vertically, entrepreneurs exchange money for the hire of new factor inputs; viewed horizontally, entrepreneurs buy and sell intermediate goods and services as the production cycle advances towards completion. In effect, with each horizontal sale of a completed intermediate good or service a vertical payment to the factor owners who participated at that stage of production is made. In contrast, at the end of each production cycle, entrepreneurs sell their output to the factor owners whose inputs have just been remunerated at the same  $t$ , but in the production of yet-to-be-realized output of concurrent production cycles.

As the cost of worn capital is included among these factor payments, the owners of capital are paid just as regularly as are the owners of other factor inputs. Important to note is that these latter payments are a return on pre-existent capital stock and do not represent return on new capital stock that is being formed.<sup>8</sup> Although no numerical distinction between new capital formation, for which no reward is received until the end of a production cycle, and replacement and operating capital for which payment is made at each stage of a production cycle, it is still important to distinguish between these two.<sup>9</sup> Accordingly, the length of a production cycle can never be shorter than the time it takes to realize the investment begun at the beginning of the production cycle, and can never be longer than the time necessary for new capital formation to yield its first return. The length of the production cycle is crucial in the calculation of the total cost of capital and its realized -- not necessarily promised ( $r^e$ ) -- real rate of return ( $r$ ).<sup>10</sup>

In summary, factor owners, including those of pre-existent capital, are always compensated in the moment of their sacrifice -- the real value of their compensation being that of the output of the concurrent production cycle just ending in the moment of their payment. With respect to factor owners in particular, the number of real goods and services available at a given  $t$  is determined by the output of the concurrent

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<sup>8</sup> Intermediate goods, although surely a form of capital stock, are not what is intended here. Indeed, it is the expended fixed capital (plant, machinery, and equipment) that contributes to the production of intermediate goods and services that is meant.

<sup>9</sup> The promise made to investors on the part of entrepreneurs is reflected in an expected rate of return ( $r^e$ ) to which entrepreneurs and their investors agree in the market for new investment at the beginning of each new production cycle ( $t_0$ ). This investment can be utilized in three very ways. One, it pays for the factor inputs necessary to maintain the fixed capital stock -- plant, machinery, and equipment -- of previous investments. Two, it pays for the factor inputs necessary to form new capital that is formed for the first time. Whereas the return on old fixed capital is already known and not expected to change -- although it could -- the return on new capital formation is unknown and presents a much greater risk to both investors and the entrepreneurs whom they fund. Three, it pays for the operating (or working capital) necessary to cover temporary short-fall at the end of concomitant production cycles whose output in real goods and services are purchased by factor owners as their inputs are expended.

<sup>10</sup> This point will be taken up again when we discuss the market for new investment.



production cycle just-ended at  $t$ . The extent, if any, to which this amount depends on the current production cycle will be discussed later.

If factor owners are not paid, they will cease providing their inputs. In contrast, if investors are not remunerated what they were promised they suffer a loss that effects their willingness to invest more. Much can change between the beginning and the end of a given production cycle, and the amount paid to investors at the cycle's end may or may not be the amount promised (or expected) at its beginning -- namely,  $r^e$ . Entrepreneurs are constrained in that they can never pay more than what they actually earn without suffering a loss or raising still more funds in the investment market. But, what investor will want to invest, if an entrepreneur has failed to make good on his previous projects?

How much an entrepreneur is able to pay at  $t_1$ , and how much he earns over the period  $t_0$  to  $t_1$  is the subject matter of the following discussion.

#### THE COST OF CAPITAL AND THE REAL RATE OF RETURN

Returning to equation 1 with hopefully a clear understanding of the costs and payments that it represents we can now rewrite it as the value of final output of the just-completed production cycle under consideration.

$$Q_1 = V(t_1; I_0) \quad (\text{equation 3})$$

where

$t_1$  = the end of the production cycle  
 $I_0$  = the amount of capital invested at its outset  
 $Q_1$  = the value-added cost of the invested outcome

The area under the curve is captured by the integral

$$K_1 = \int V(t; I_0) \quad (\text{equation 4})$$

evaluated over the interval  $t_0$  to  $t_1$ . The constant normally associated with the valuation of an integral from its derivative has been set to zero.  $K_1$  represents the total value of invested capital at  $t_1$  based on an initial investment of  $I_0$  made at  $t_0$ . It is the time-value of capital employed during the course of the current production cycle just-ended. If no investment other than that required to maintain preexistent investments is made, then  $K_1$  is not expected to change from one production cycle to the next.<sup>11</sup> To be sure, equation 4 is much more than a simple accounting fiction; it includes the value of all factor inputs that have been bound in time from the initial point of investment ( $t_0$ ) until the delivery of final output ( $Q_1$ ) at  $t_1$ . The time-value of capital may be best construed as the opportunity cost of capital, for capital bound to the current production cycle is capital that cannot be used in other concurrent production cycles without interrupting the

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11 Once again, the capital required to maintain preexistent investments consists of both replacement capital and working capital. The former represents the cost of depreciation and upkeep of fixed capital that is worn during the production process; the latter represents capital set aside to insure an even flow of payments to the owners of factor inputs as the production cycle from one stage of production to the next.

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current production cycle. It includes all forms of new capital formation including new fixed capital and intermediate goods. In this context replacement capital and operating (working) capital are treated just as any other factor input insofar as they contribute only to the creation of new fixed capital and intermediate goods. Always keep in mind that the goal of production is finished output that can be consumed by factor owners and their dependents, and that everything produced in between is waste, if it cannot result in this fundamental economic end. The time value of capital is not a measure of total capital wealth! For, if it were, it would be much larger in value and could just as easily represent eventual economic waste or scrap as assets with as-yet-unused, potential value in production. Indeed, the time-value of capital measures only that amount of total wealth actually employed in the production of  $Q_1$ . Accordingly, although it includes the cost of formation of new capital initiated at  $t_0$ , it does not include the cost of capital formation of pre-existent capital; rather, only that portion of pre-existent capital that is actually expended. This would include things like depreciation and maintenance costs. Intermediate goods from other concurrent production cycles are not included, as these are formed and used in the production of the output of the cycles in which they were produced.<sup>12</sup>

The longer an input is tied up in production -- i.e., the longer the intermediate good to whose production it contributed remains in a state of transformation -- the more valuable it becomes as a source of cost. In figure 1 the shaded area under the value-added cost function from  $t_0$  to  $t$  represents the time-value (or opportunity cost) of all inputs employed from  $t_0$  to  $t$ . All of this value represents a liability to the entrepreneur and his investors until it has finally reached  $t_1$  and can be sold to factor owners and other consumers as finished goods and services. In this sense,  $K_1$  is not a flow, but a stock against which we can measure a flow -- namely, the finished goods and services ( $Q_1$ ) sold to factor owners and their dependents at the end of the cycle. As a result, the real rate of return on employed capital of the current production cycle just-ended becomes

$$r_1 = Q_1/K_1 \quad (\text{equation 5})$$

In other words, the real rate of return ( $r_1$ ) is equal to the value-added cost of the finished *delivered* product ( $Q_1$ ) divided by the opportunity (time-value) cost of all inputs that went into its production ( $K_1$ ).<sup>13</sup> This relationship is of particular importance, no matter the shape of  $V(t; I_0)$ .<sup>14</sup>

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12 It is possible that intermediate goods produced in a concurrent production cycle that fails to meet its scheduled output are used in the current cycle, but this changes little insofar as the owners of the goods have to be compensated in the same way that all owners of factor inputs are rewarded -- namely, with money income used to purchase the finished output of the concurrent cycle-just-completed at the time of input.

13 If this output is not delivered, then it is not sold and cannot contribute to the value of  $Q_1$ , only the cost of  $K_1$ . Thus, the large numbers of unloaded, but not yet delivered automobiles that fill dockyards around the world in times of global economic downturn are not finished output. Rather, they represent intermediate goods -- i.e., capital cost -- that could just as easily become economic waste as finished output, if they are allowed to sit indefinitely. Indeed, the owners of the land must be paid rent. In other words, the cost mounts both horizontally and vertically across time.

14 So far we have only required that the function,  $V(t; I_0)$ , be monotonically increasing across time (see equation 2a). There is no compelling reason -- other than mathematical convenience -- for the resulting curve to be shaped in the manner depicted in figures 1 and 2. See Appendix 1..

### The Model's Components

Firstly, we observe that  $Q_1/K_1$  is of the form  $(1/K)(\delta K/\delta t)$  evaluated at  $t_1$ . From this relationship we can write

$$rt = \log K(t; I_0) \quad (\text{equation 6})^{15}$$

and note that differentiating both sides of equation 6 with respect to  $t$  yields

$$r = \delta(\log K(t; I_0))/\delta t = (1/K)(\delta K/\delta t) \quad (\text{equation 7})$$

This implies that equation 6 is of the form

$$K(t; I_0) = e^{rt} \quad (\text{equation 8})$$

The relationships between  $K$  and  $t$  and  $K$  and  $I_0$  are similar in sign, but different in magnitude from those of  $V$  and  $t$  and  $V$  and  $I_0$ .

$$\delta K/\delta t > 0 \quad (\text{equation 9a})$$

$$\delta K/\delta I_0 > 0 \quad (\text{equation 9b})$$

Notice that the real rate of return of the production cycle subsequent to the one just completed rises, if no new capital formation -- i.e., capital formation other than that of intermediate goods production and replacement and operating capital -- takes place during the cycle. This is because the value of  $K_1$  in equation 5 falls relative to  $Q_1$  as only replacement and operating capital are necessary to achieve the same, or hopefully, higher level of output. Indeed, what before was newly formed capital has now become pre-existent capital and only its replacement (depreciated) value enters into equations 3 and 4. Let us consider this scenario in greater detail, for it will help us to understand better the relationship between  $Q_1$  and  $K_1$  and the comparative statics that will later result.

During the production cycle just-ended, the values of  $Q_1$  and  $K_1$  rise, because of the increase in new capital formation ( $I_0 > \text{the cost of replacement and operation of existent capital}$ ). However,  $K_1$  rises more than  $Q_1$  because it is subject to time-value that  $Q_1$  is not. This has the effect of lowering the real rate of return  $r_1$ . During the subsequent production cycle, the value of new investment is now lower than what it was at the beginning of the production cycle just ended, but higher than the end of the production cycle antecedent to the one just completed. This is because new productive capital was formed during the just-completed production cycle, and its use during all subsequent production cycles implies a higher cost of depreciation on total employed capital. In other words, there is now more capital in the system to wear and therefore more investment is required to replace it. As a result  $K_1$  also rises above what it was during the production cycle antecedent to the production cycle just-completed, but falls significantly in relationship to  $K_1$  of the cycle just completed, as the cost of new capital formation has disappeared. This is especially true insofar as new capital formation always takes place toward the beginning of a production cycle, and the time-value of capital formed at the beginning of a production cycle is at its greatest towards the end of the cycle in which it was formed.<sup>16</sup> We will return to equations 7, 8, and 9 later in our discussion of the demand

15 The symbol  $\log$  stands for the natural log. In science and mathematics it often appears as  $\ln$ .

16 It probably does not hurt to recall that  $K_1$  includes the time-value of all factor inputs -- not just those employed in the forma-



for new investment. For the moment let us return to equation 5 from which we can derive a quantifiably workable form of Roger Garrison's version of Hayek's triangle.

### MODIFIED HAYEK'S TRIANGLE

Metaphorically speaking, we can derive Hayek's Triangle (figure 3) from figure 1 by reversing the direction of the x-axis in figure 1, draining the area under the value-added cost function, and plotting this drained value along our newly constructed horizontal axis. The vertical axis remains in tact, and the market value of total output at  $t_1$  -- namely,  $Q_1$  -- remains unchanged.<sup>17</sup>

Unlike figure 1, but like Garrison's interpretation of Hayek's triangle, the area of the triangle contains no meaningful information. Accordingly, our interest in the triangle lies only in its height  $Q_1$ , its length  $K_1$ , and the slope of the triangle's hypotenuse -- namely, the real rate of return on real employed capital evaluated at  $t_1$ . In general we will only be very interested in  $t$  at the end and beginning of the production cycle under consideration

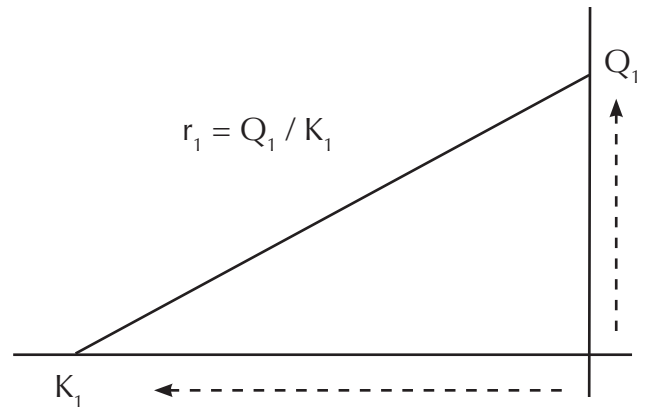


Figure 3. Hayek's Triangle

-- hence, the use of the integer-subscripts instead of the naked variable  $t$ . The point where the vertical and horizontal axes intersect is, of course, fixed, but any change in the value of  $K_1$  is now measured from right to left. To the extent that  $(t_1 - t_0)$  is measurable along the horizontal axis it is only in terms of its effect on the accumulated value of the opportunity cost of real capital measured by the area under the curve in figure 1. Thus, in the absence of any change in new investment at  $t_0$ , an increase in the length of the production cycle will cause  $K_1$  to increase and the base of the triangle to lengthen. Accordingly, a decrease in the length of the production cycle will have just the opposite effect (see equation 9a). We should also note that the base of the triangle lengthens when there is an increase in new investment beyond that of mere replacement and operating capital, and that it shortens in the absence of new capital formation and a short-fall in replacement capital. Finally, in the unlikely event of no change in new investment when new technology is introduced the base of the triangle will shrink.

The triangle depicted in figure 2 is not the same as Hayek's triangle, but it does capture his crucial notion of the time-value of capital and prepares, hopefully, the way to a better understanding of the essential role of interest rates in the temporal coordination between current sacrifice (real savings) and future consumption (real investment). So as to be clear about this difference, Hayek depicted only time along the horizontal axis and only final consumption along the vertical axis -- hardly a very useful recipe for the construction of a meaningful mathematical model of general equilibrium, and certainly a source of much confusion

tion of new capital and the maintenance and operation of old. In this sense, intermediate goods are included as a part of capital formation, but do not represent capital that produces a return beyond the production cycle in which they are formed.

17 Unlike Garrison's interpretation of Hayek's triangle, however, the value plotted along the vertical axis is not the value of total consumption; rather it is the total value-added cost of output.

and obstruction when the triangle was first presented. Hopefully, this model will provide a better heuristic tool for the understanding of Hayek's profound insight with regard to the nature and formation of business cycles and the important role that inflated money supplies play in their creation. With this in mind, we should not confuse the just-described production cycle with what is typically thought of as a business cycle today. For, in a real economy in which interest rates provide an efficient price mechanism for the temporal coordination of sacrificed current consumption (real savings) and enhanced future consumption (real investment) there is no business cycle -- only regularly occurring, ever adjusting, production cycles. Indeed, the intended purpose of this model is to show that business cycles, typically characterized by the irregular reoccurring boom-bust phenomenon, are unnecessary and avoidable distortions of otherwise generally well-functioning free-market production cycles.

## Time-Preference and the Natural Rate of Return

It is common in the economics literature to speak of natural rates, as if there were givens toward which economics systems gravitate. Even with the assumption of unlimited want, in a world of economic scarcity the extent to which we can develop our resources is limited by our level of technological sophistication. Technology is not the only constraint, however, for the time preference of the individual varies from individual to individual and society to society, and when it is very high we are little more than simple hunters and gathers roaming near and sometimes very far from our next meal. In his work, *Democracy: The God that Failed*, Hans-Hermann Hoppe expounds masterfully on the importance of time preference and the role of private property in the advancement of civilization and the role of the state in its devolution.<sup>18</sup> The following section treats this notion of time preference mathematically and as a given in the investors decision to consume now or invest in his future consumption. Obviously, it stretches the imagination to assume that everyone's time preference is the same. Then too, those with a lower preference for current consumption (low time-preference) are likely to have command over far greater wealth than those with a higher preference. As a result, the former are likely to form a more homogeneous group and play a greater role in the determination of how finished goods and services are allocated across time in a given society.

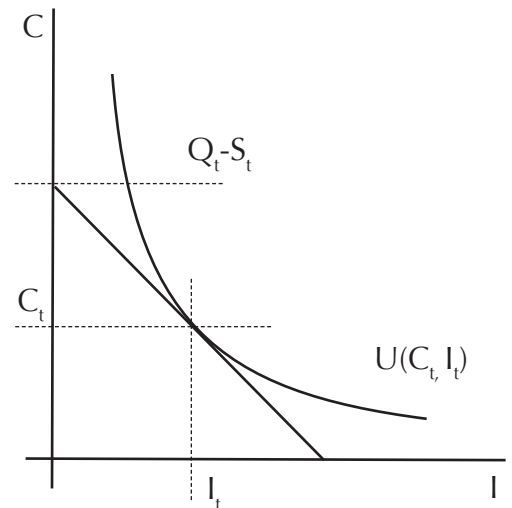


Figure 4. Utility Maximization Problem

### CURRENT AND FUTURE CONSUMPTION TRADE-OFF

This model assumes that the consumer is faced with three alternatives: he can consume ( $C_t$ ) now, set aside ( $S_t$ ) for consumption at a later date, or invest ( $I_t$ ) in an effort to elevate his current level of consumption and savings to an even higher level in the future.<sup>19</sup> What he cannot do is increase his level of current

<sup>18</sup> Hoppe. 2001.

<sup>19</sup> Although it is not difficult to insert government into this model, for the sake of simplicity and a clear understanding of the

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and future consumption simultaneously without a corresponding increase in his level of income ( $Q_t$ ), or alternatively, a diminishment in his current level of saving ( $S_t$ ). In this model savings ( $S_t$ ) are an alternative to investment ( $I_t$ ) and not a prerequisite for this latter's occurrence. Indeed, investment represents in this component of the model represents little more than a desire on the part of investors for greater future savings and consumption -- a desire that must be negotiated in the market place with those who provide the ideas, industry, knowledge, and talent to satisfy it -- namely, the entrepreneur.<sup>20</sup> In general, the trade-off between current and future consumption is consistent with the fundamental economic notion that nothing comes from nothing, but ideas, that in and of themselves require some sacrifice in time and resources. The trade-off, then, is captured in figure 3 by the negatively sloped budget constraint on the one hand, and the convex level curve of the quasi-concave utility hill ( $U_t$ ) for current consumption ( $C_t$ ) and the investment ( $I_t$ ) required for increased future consumption on the other hand. The constrained maximization problem that results from this utility function and its corresponding budget constraint form the basis for the natural time-preference of the economy under consideration.

Certainly, there is much to say that would reflect poorly on such a theoretical construct. Indeed, not all consumers are the same; and surely, not all consumers are even investors, let alone factor owners. What is more, wealth is not uniformly distributed among any nation of people (or more likely peoples), and the time preference of those with much wealth is surely different from that of those with little or no wealth. Obviously, this model cannot accommodate these blatant disparities. This said, no model that explains everything has ever been very useful at explaining anything. So, let us focus on what the model does explain.

Obviously, economic growth is not merely a matter of desire, but in the absence of desire, how could entrepreneurs, that are not security-monopolists with the authority to counterfeit and tax, ever gather the real goods and services required to pay factor owners for their sacrifice? Indeed, this model assumes no worse than a well-intentioned national planning board that imposes its own set of preferences on everyone. In fact, it offers much better, for the imposition goes no further than the assumption of the existence of a single temporal preference set for an entire economy. Furthermore, we do not even need to know what this preference set looks like; rather, we assume only that it exists and that a natural expected real rate of return can be derived from it.<sup>21</sup>

### THE BUDGET CONSTRAINT

Output ( $Q_t$ ) in this component of the model is the same as that of the previous component. It is the market value received by the owners of factor inputs at the end of one production cycle and the beginning of another. Further, as we are dealing with a unitary good that has multiple uses we can assign a separate price to each of those uses just as we would completely different goods. Our initial formulation thus becomes

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model's primary objective, we will leave it out. The same can be said for not wishing to open the economy to external trade.

20 This negotiation forms the third component of this model and will be introduced in a later section.

21 Beyond quasi-concavity of the preference hill, the precise shape of the hill is relatively unimportant. In an open trading economy we would likely take a greater interest in the shape of the hill, but we must reserve such consideration for a later paper.

The Model's Components

$$P_0 Q_t - (P_1 C_t + P_2 S_t + P_3 I_t) = 0 \quad (\text{equation 10})$$

and where<sup>22</sup>

$Q_t$  = national private sector output/income  
 $C_t$  = national private sector consumption  
 $S_t$  = national private sector savings  
 $I_t$  = national private sector investment

Dividing equation 10 through by  $P_0$  and assigning numéraire status to our unitary output  $Q$  obtains

$$Q_t - (P_1 C_t + P_2 S_t + P_3 I_t) = 0 \quad (\text{equation 11})$$

Under the further assumption that savings are simply an alternative form of current consumption with no special price of their own, we can further set  $P_0 = P_1 = P_2 = 1$ . This leaves only  $P_3$  as a relative price worthy of our further attention -- well, at least for the moment. Further, and for the sake of notational convenience, let us drop the subscript  $t$  and recall in so doing that each and every  $t$  represents both the end of one production cycle and the beginning of another no matter the length of either. Indeed, our budget constraint further simplifies to

$$Q - (C + S + P_3 I) = 0 \quad (\text{equation 12})$$

All values in the constraint are considered non-negative.

Before moving on to the nature of  $P_3$ , let us pause and consider more thoroughly the nature of real money, real savings, and real wealth.

### THE NATURE OF SAVINGS, MONEY, AND INVESTMENT

In the sense that savings are treated as an alternative use of money income at the end of each production cycle and thus constitute an immediate use of each remittance received by factor owners, savings contribute to the stock of uninvested wealth. This stock can be held in various forms, but for the purpose of this model, only two need to be considered very carefully: illiquid stock including real goods such as foodstuffs, clothing, housing, and other forms of property; and liquid stock -- namely, money -- that is used in exchange for illiquid stock when the need arises. In this sense, money is always a *traded stock*. For, in the moment that it is surrendered in exchange for some good or service it becomes the stock of the seller. In a real closed economy, there is no market for the supply and demand of money, except at its source, as money is generally not consumed.<sup>23</sup> Its value is everywhere determined by what it can obtain in the market place in exchange for other goods and services.<sup>24</sup>

22 The epithet national private sector is used to remind the reader that we are dealing with a closed economy in the absence of a state-security monopolist, and that government (the monopolist) can be easily added at a later point.

23 An exception to this might be the presence of two different, freely traded objects -- say, gold and silver -- that both serve as money and whose value changes with the relative supply of each.

24 Although some people may have a greater preference for liquid over illiquid stock, the amount of liquid stock does not

## The Model's Components

Further, we must be careful to distinguish between the savings of firms and those of households. Although it is true that firms can store money and maintain inventories, these latter cannot be treated the same as household stock. Industrial inventories represent cost, pent-up capital, that is bound in production. Household savings are decidedly different in this regard, as they represent finished goods that are fully consumable and not bound to any production process for their value. Even finished goods that are held by producers cannot be considered savings, for such goods are just another form of inventory that may or may not make it past the final stage of production -- namely, retail sale and delivery. Indeed, it is by no means certain that finished goods inventory will ever be sold and become a finished good or service. Finally, this is not to say that households cannot store finished goods that they will never use. These too, can be a form of waste, but they are not a form of inventory in the production sense and cannot be treated as a capital expense.

In contrast, retained earnings are liquid holdings that can be used to purchase finished goods and services desired by the owners of factor inputs. As such, these earnings can be used for new investment and can be rightfully treated as a portion of savings.

Finally, we could easily include savings as a decision variable in our utility function, as it is surely the case that people have a preference for how much of their income they set aside as an alternative to consumption and new investment. This said, there is no reason to assume more than we must, and it is highly unlikely that everyone's preference for savings is the same. What is more by leaving savings as an exogenously determining variable, we can accommodate Keynesian economists who believe that people tend to hoard in times of crisis.

Further, so long as money is sound -- a tradable commodity, or equally valued and tradable paper substitute, not subject to currency inflation --, it promises both security and the possibility of a positive return.<sup>25</sup> This makes money a very real alternative to the risky nature of new investment, and like consumption savings are priced more highly than investment in the moment of sacrifice.

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change unless more of it is produced. What can and does change is the way in which the liquid stock is distributed throughout a population. If the amount of liquid stock does not keep pace with an increase in illiquid stock, then a temporary shortage occurs, and its value relative to the other goods and services for which it is exchanged rises. This increase in value encourages the production of more liquid stock (real money), and its supply increases. In the long run then, there will never be too much or too little money, there will only be a change in its value relative to the value of other goods and services. If its value become so great relative to other goods and services that it can no longer be used to make small purchases, then it can be broken down into smaller pieces or used to purchase some other good -- e.g., gold for silver -- that is more plentiful and would make for a better supply.

25 The increase in the value of savings occurs because advancing technology increases productivity and consequently the value of a nation's productive assets. Unless the money supply is increasing faster or at a rate approximately equal to productivity increases, the value of money is likely to rise with the value of the assets against which it is traded. It must be kept in mind, that when money is sound, it is subject to all of the vagaries of other commodities, but by its special nature as a medium of exchange for all goods and services its value is likely to be more stable. In effect, money is just another good held as inventory, but of a highly liquid nature. It is its scarce and liquid (it can be traded for almost anything) nature that makes it more valuable than any of its other uses as a non-monetary commodity. Thus, we can speak of a money-premium.



### THE PRICE OF FUTURE CONSUMPTION AND NEW INVESTMENT

Were we to plot investment against consumption in the same way that we might plot savings against consumption we would obtain a straight line with a slope of -1. Although, this would surely constrain our level of utility and provide the basis for a utility maximization problem, it would be of very limited use and not permit the modeling of time preference in a meaningful way. After all, no one can know with certainty what the future will bring, and our expectations with regard to the future and what the future truly brings are forever in a state of adjustment. As a result, we typically price current and future consumption differently.

Unlike the factor owner who is paid and consumes as he goes along, the investor is not rewarded for his sacrifice until his investment is realized by the entrepreneur. Furthermore, and for many, more importantly so, the investor must bear the risk of his sacrifice never coming to fruition. Thus, the investor expects a reward for his sacrifice that is higher in value than what is actually sacrificed in the moment of surrender. One way to express this difference in value is given in equation 13a. The amount of expected additional value ( $C_t r^e$ ) is represented as a decimal fraction of what is actually sacrificed in the moment of surrender.

$$C_t (1 + r^e) = I_t \quad (\text{equation 13a})$$

Dividing both sides of equation 13a by  $(1 + r^e)$  yields the price of new investment as a function of the numéraire good  $C_t$ -- alternatively,  $Q_t$  or  $S_t$ .

$$P_3 = 1/(1 + r^e) \quad (\text{equation 13b})$$

or 
$$C_t = P_3 I_t \quad (\text{equation 13c})$$

It should be clear that current consumption ( $C_t$ ) is at all times dearer to the investor than what he hopes to receive in return for his investment ( $I_t$ ) in the moment of sacrifice. Clearly, it takes more units of  $I_t$  to equal one unit of  $C_t$ . Finally, if the investor's sacrifice is properly rewarded, then it will be at the end of the production cycle at whose beginning the sacrifice in current consumption is made.

As before we once again drop the subscript  $t$ . Rewriting equation 12 in terms of  $C$ , we obtain a formula for the line that represents our budget constraint.

$$C = (Q - S) - P_3 I \quad (\text{equation 14})$$

Notice that  $Q - S$  is the intercept of this line and that  $-P_3$  is its slope. Further note that an increase in output shifts the budget constraint upward, and that an increase in savings pushes it downward with zero effect on the relative price of  $C$  to  $I$ . Further, an increase in the expected real rate of return ( $r^e$ ) causes the absolute value of  $P_3$  to fall, and the budget constraint to rotate outward with the intercept as its point of leverage. This, of course, increases the vector space available for a possible solution of the utility maximization

## The Model's Components

problem. A fall in  $r^e$  has the opposite effect.

Finally, it should be mentioned in passing that the value of the expected real rate of return ( $r^e$ ) in equation 7 and the value of the real rate of return ( $r_t$ ) in equation 4 are derived differently and may or may not be equal. This said, they must be equal in order for general equilibrium to occur, but we still have some ways to go, before we can properly address this point. Let us now turn to the utility function.

### THE UTILITY FUNCTION

The utility function is given by equation 15 and assumes all of the properties that are common to preference hills in the economics literature.

$$U_t = U(C_t, I_t) \quad (\text{equation 15})$$

where  
and where

$U_t$  = national utility  
 $C_t$  = national private sector consumption  
 $I_t$  = national private sector investment

Firstly, both consumption and new investment are considered economic goods insofar as more of each is better. This is the property of *non-satiation*. Though some people will argue that too much of a good thing can become a bad, we must keep in mind that additional utility can always be obtained from a good in the form of a gift from its owner.

$$\delta U_t / \delta C_t > 0 \quad (\text{equation 16a})$$

$$\delta U_t / \delta I_t > 0 \quad (\text{equation 16b})$$

Further, we assert that the principle of diminishing marginal utility.

$$\delta^2 U_t / \delta C_t^2 < 0 \quad (\text{equation 17a})$$

$$\delta^2 U_t / \delta I_t^2 < 0 \quad (\text{equation 17b})$$

Although not a set of conditions sufficient to insure an optimal solution, these are certainly conditions necessary to assure such an outcome. In practical terms they simply indicate the tendency to tire in the consumption of one good and the natural preference for consumption of many goods.

This utility function, like all utility functions in economic theory, is highly abstract and not a representation of something that can be measured in the real world. Its utility, pun intended, is in the relationships that can be derived from it. Our only concern is that it accurately expresses the ordinal nature of our consumers' subjective preferences, and that it is well-behaved as a mathematical function and thus everywhere differentiable. As real money is a fungible good that can be infinitely divided -- or at least substituted with another real good, that can be more readily broken up --. and as the good that is traded is a unitary good with different uses, our mathematical requirement of a well-behaved function is well-satisfied.

Once again, that the individual preferences of an entire nation can be aggregated into a single preference

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set, although far-fetched, is no more so than the notion of a single numéraire good for the vast number of industrial processes that go into the production of a nation's finished output. Alas, let us not seek to justify one absurdity with another; rather, let us focus on the ability of our model's component to generate a supply curve for new investment and testable hypotheses that do not omit crucial variables.

### THE MAXIMIZATION OF NATIONAL UTILITY

Let us now combine our national utility function and budget constraint and solve for the values of our decision variables. In its simplest and clearest form the maximization problem is given as follows:

$$\begin{array}{ll} \text{w.r.t.} & \max_{C \text{ and } I} U(C, I) \end{array} \quad (\text{equation 18a})$$

$$\text{s.t.} \quad Q - (C + S) - P_3 I = 0 \quad (\text{equation 18b})$$

$$\text{where} \quad Q = V(t_0, I_{-1}) \quad (\text{equation 18c})$$

Although we have eliminated the  $t$  subscripts in equations 18a and 18b, they have been retained in equation 18c. This is to distinguish between  $I$ , that is a decision variable, and  $I_{-1}$ , a known parameters of the just-completed production cycle that is antecedent to the one under consideration. In a similar light,  $t_0$  is simultaneously the end of the just completed production cycle, and the beginning of the new production cycle for which the problem of optimization must be solved. In summary, the value of  $Q$  is a parametric given based on the outcome of the antecedent production cycle.

Although it is possible to replace  $C$  in equation 18a by substituting with equations 18b and 18c, and then maximize solely with respect to  $I$ , it proves more insightful to solve the problem as the LaGrangean expressed in equation 19.

$$\begin{array}{ll} \text{w.r.t.} & \max_{C, I, \lambda} \mathcal{L} = U(C, I) + \lambda(Q - C - S - P_3 I) \end{array} \quad (\text{equation 19})$$

Differentiating with respect to  $C$ ,  $I$ , and  $\lambda$  yields the first order conditions necessary for a maximum. They are

$$\delta \mathcal{L} / \delta C = U_C - \lambda = 0 \quad (\text{equation 20a})$$

$$\delta \mathcal{L} / \delta I = U_I - \lambda P_3 = 0 \quad (\text{equation 20b})$$

$$\text{and} \quad \delta \mathcal{L} / \delta \lambda = Q - (C + S + P_3 I) = 0 \quad (\text{equation 20c})$$

Rearranging the terms of equations 20a and 20b, dividing the former into the latter, and eliminating  $\lambda$  obtains

$$U_I / U_C = P_3 / P_1 = P_3 \quad (\text{equation 21a})$$

$$\text{or} \quad \delta C / \delta I = P_3 / P_1 = P_3 \quad (\text{equation 21b})$$

Whereas equation 21a provides the economic interpretation of the conditions necessary for a maximum,

## The Model's Components

equation 21b provides the diagrammatic interpretation of the same. In short, the market value of the utility gained from an increase in an increment of new investment, must match exactly the market value of the utility lost from the corresponding incremental decrease in current consumption. In effect, the solution for our optimization problem is the point of tangency ( $C^Q$ ,  $I^Q$ ) between our budget constraint and a level curve of our utility function depicted in figure 3.

From equations 20a, 20b, and 20c we solve for  $C^Q = C(Q, S, P_3)$ ,  $I^Q = I(Q, S, P_3)$ , and  $\lambda^Q = \lambda(Q, S, P_3)$ , substitute back, and obtain the identities provided in equations 22a, 22b, and 22c. From these we can derive the second partials, a demand function for new investment, and its anticipated sign. The Q superscript of the identities reminds us that the optimal values for C, I, and  $\lambda$  must lie within the vector space created by Q at the end of the previous period's production cycle.

$$U_C(C^Q, I^Q) - \lambda^Q = 0 \quad (\text{equation 22a})$$

$$U_I(C^Q, I^Q) - \lambda^Q P_3 = 0 \quad (\text{equation 22b})$$

$$\text{and} \quad Q - (C^Q + S + P_3 I^Q) = 0 \quad (\text{equation 22c})$$

### THE INVESTORS' DEMAND FUNCTION

Differentiating each of our identities (see equations 22a, 22b, and 22c) with respect to  $P_3$  yields a system of equations that can be described by a 3X3 bordered Hessian matrix, a 3X1 vector of unknowns, and a 3X1 vector of constants. From the determinant of this bordered Hessian we can derive the second order conditions sufficient for a point of maximization at ( $C^Q$ ,  $I^Q$ ) and determine, where possible, the sign (testable hypothesis) of each of our unknowns -- namely,  $\delta C^Q / \delta P_3$ ,  $\delta I^Q / \delta P_3$ , and  $\delta \lambda^Q / \delta P_3$ .<sup>26</sup>

Without providing a diagram of our bordered Hessian the value of the determinant so described is given by

$$D = U_{CC} D_{11} + U_{IC} D_{21} + (-1) D_{31} \quad (\text{equation 23})$$

for which  $D_{11}$ ,  $D_{21}$ , and  $D_{31}$  are the signed cofactors of D, and  $U_{CC}$ ,  $U_{IC}$ , and -1 are the second partials of our LaGrangean equation optimized with respect to C, I, and  $\lambda$ , respectively. Expanding the cofactors and applying the appropriate algebra yields the following sufficient condition for a point of maximization.

$$-U_{CC} P_3^2 + 2U_{CI} P_3 - U_{II} > 0 \quad (\text{equation 24})$$

By assumption  $U_{CC}$  and  $U_{II}$  are both negative (see equations 17a and 17b), thus it must be true that  $U_{CI}$  is positive, else a point of maximization is not assured.

$$U_{CI} = U_{IC} > 0 \quad (\text{equation 25})$$

<sup>26</sup> The requirement for a point of maximization is that the border preserving principal minors of the bordered Hessian's corresponding determinant alternate in sign. The formula for establishing the order of alternation is given by the following:  $(-1)^{m-r}$ ,  $m = 2r + 1 \dots n + r$ . The letter m is the dimension of one side of a square determinant whose sign must prevail. The letter r refers to the number of constraints (borders), and the letter n is the number of variables for which we eventually solve. As our bordered Hessian is only a 3X3 matrix, only one principal minor results. In effect,  $(-1)^{3-1} = 1$ , and  $2r + 1 = n + r = 3$ .

### The Model's Components

We can now apply Cramer's rule and solve for  $\delta I^Q / \delta P_3$ . This yields the slope of our investor's demand function ( $I^Q = I(P_3; S, Q)$ ) for new investment at the point of maximization.<sup>27</sup>

$$\delta I^Q / \delta P_3 = I^Q D_{22} = -\lambda^Q < 0$$

or simply

$$\delta I^Q / \delta P_3 < 0$$

(equation 26)

It can be shown from our first order conditions that  $\lambda > 0$  for all  $\lambda^Q$  that are a solution to our maximization problem. In other words, the demand for new investment on the part of investors is inversely related to the price of new investment -- a mathematical outcome well in keeping with observed economic behavior. Solving for the cross-partial  $\delta C^Q / \delta P_3$  yields an equally conclusive outcome -- namely,

$$\delta C^Q / \delta P_3 = -\lambda^Q D_{21} = \lambda^Q P_3 > 0$$

or simply,

$$\delta C^Q / \delta P_3 > 0$$

(equation 27)

As before, our mathematical outcome corresponds with typically observed economic phenomenon -- namely, that a rise in the price of a substitute good increases demand for that good's substitute.

Although there are surely many other similarly important results that we can derive from similar analysis, we still have much to consider before we have fully constructed our general equilibrium model and must proceed to the third and final component of our general equilibrium model.

## The Price Mechanism for Temporal Coordination

Although it is frequently referred to in the economics literature as the *loanable funds* market, it would be wrong to use this term in the context of this model. For, any market mechanism that can provide for the voluntary transfer of real finished goods from investors to factor owners across time via a third-party entrepreneur is a good candidate for this modular component. Whether entrepreneurs sell stocks, issue bonds, or borrow from a bank is largely irrelevant so long as he is able to provide investors with their expected return, remunerate factor owners, and maintain sufficient control over this transfer of wealth and the inputs that he employs. What the entrepreneur brings to the table in this transaction is the industry, knowledge, acumen, and foresight to turn existent finished goods and services into productive capital that results in the creation of even more finished goods and services -- in short, the diversion of existing inputs from lower to higher valued uses. This is the mark of a true entrepreneur in a real economy; it is also a very far cry from what many entrepreneurs and investor engage in today. Let us turn first to the supply-side of

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<sup>27</sup> This is merely a reformulation of the expression  $I^Q = I(Q, S, P_3)$  obtained as one of the solutions to our first order conditions (see equations 20a, 20b, and 20c). It holds  $S$  and  $Q$  constant while allowing  $P_3$  to vary.



the investment market where we already enjoy a giant forward step ahead.

#### SUPPLY OF NEW INVESTMENT

In light of the current state of the economics profession, especially among those who dwell in the ethereal world of *Keynesian sociology*, it cannot be said enough: *real* investment cannot succeed in the absence of *real* sacrifice. All else is either folly, theft, or knowledgeable deception. The owners of factor inputs must be paid with real goods and services, and when the money that they receive cannot buy these goods and services there is trouble. Further, it is only because investors forego their own consumption in real goods and services that there is something there for the owners of factor inputs to purchase. Thus, when entrepreneurs and investors come together, they negotiate over the price and amount of wealth to be transferred across time. This negotiation is based on the current value of factor inputs and outputs -- value that is expected to remain relatively constant or at least largely predictable across the length of the production cycle. Indeed, in a constant growth real economy that outpaces the production of new money, the value of money relative to other real goods and services would actually rise. As a result, the prices of factor inputs and produced output would fall over time.

Important to keep in mind is that factor owners are paid as their inputs are employed, and that no concurrently running production cycle excludes another. The decision variables  $C^Q$  and  $I^Q$  determine only how the goods and services just finished at the end of one production cycle will be allotted at the beginning of the new production cycle. To the owners of factor inputs neither does it matter in which cycle his inputs are employed and his remuneration is received, nor does it matter at what point in any particular production cycle that his purchases are made. Where a production cycle begins and ends is not a concern for him, as he stands ready to supply his inputs at any time that he can receive payment. It is in this regard that he differs fundamentally from both investors and the entrepreneurs.

From our first order conditions in equations 20a, 20b, and 20c we derived the investors' demand function for new investment which we characterized as

$$I^Q = I(P_3; S, Q) \quad (\text{equation 28})$$

$$\delta I^Q / \delta P_3 < 0 \quad (\text{equation 26})$$

For the moment we will continue to ignore the relationships  $\delta I^Q / \delta S$  and  $\delta I^Q / \delta Q$ .

We know from equation 13b that  $P_3 = 1/(1 + r^e)$ . As  $r^e$  is treated as a parameter in equation 28, we also know that

$$\delta I^Q / \delta r^e = (\delta I^Q / \delta P_3)(dP_3/dr^e)$$

such that  $dP_3/dr^e = d(1/(1+r^e))/dr^e = (-1)(1+r^e)^{-2} < 0$

Since we have already shown  $\delta I^Q / \delta P_3 < 0$  we may conclude that

$$dI^Q/dr^e > 0 \quad (\text{equation 29})$$

## The Model's Components

In effect, the new investment market's supply function is simply a reformulation of the investor's demand function for new investment -- namely,

$$I^S = I(r^e; S, Q) \quad (\text{equation 30})$$

such that  $\delta I^S / \delta r^e > 0$  (equation 31a)

As  $r^e$  enters into the investor's demand function only as a parameter of  $P_3$ , any results that we obtain for  $\delta I^Q / \delta S$  and  $\delta I^Q / \delta Q$  will also hold true for  $\delta I^S / \delta S$  and  $\delta I^S / \delta Q$ . Differentiating our identities with respect to  $S$  and  $Q$  and applying Cramer's rule as we did above yields the following additional relationship for our investors' supply function

$$\delta I^Q / \delta S = (1)D_{32} / D = (-1)(-P_3 U_{CC} + U_{IC}) / D < 0$$

or  $\delta I^S / \delta S < 0$  (equation 31b)

and  $\delta I^Q / \delta Q = (-1)D_{32} = (-1)(-1)(-P_3 U_{CC} + U_{IC}) > 0$

$$\delta I^S / \delta Q > 0 \quad (\text{equation 31c})$$

It should be clear from inequalities 31a, 31b, and 31c that an increase in  $r^e$  results in upward movement along the supply curve for new investment, that given  $r^e$  an increase in  $S$  results in a leftward shift, and that an increase in  $Q$  necessarily results in a rightward shift. Further, that in all cases the exact opposite occurs when a decrease in the affected parameter is observed. None of these relationships should be at all surprising as investors are eager to invest more when they can realize a higher rate of return for their sacrifice, invest less when they choose savings over both consumption and new investment, and invest more when there is more of everything to go around.

Before turning to the demand function of the entrepreneur, let us examine more closely the nature of  $Q$  and recall the two parameters that define it -- namely,  $t_0$  and  $I_{t-1}$ . Although changes in either of these effects the level of  $Q$  and hence the level of new investment, their effect on investor demand (alternatively supply of new investment) is limited to their effect on  $Q$ . For example, if we want to know what effect a change in the previous period's level of new investment will have on the current period's supply of new investment we have only to perform the following

$$\delta I^S / \delta I_0 = (\delta I^S / \delta Q)(\delta Q / \delta I_0)$$

As we already know the signs of both  $\delta I^S / \delta Q$  and  $\delta Q / \delta I_0$  from equations 31c and 1, respectively, we can readily determine the sign of  $\delta I^S / \delta I_0$  to be positive.

### DEMAND FOR NEW INVESTMENT

Unlike the supply function for new investment that we obtained from the demand function of the utility maximization problem, we cannot derive a demand function from the information given. Indeed, all

### The Model's Components

we have is a cost function that could be easily minimized by simply not investing. Obviously, this is not the goal of the entrepreneur. Surely his first motivation is to make good on his promises to investors, for without their help no entrepreneurial activity could be undertaken in the first place. After making good on his promises to investors the entrepreneur is, of course, free to do whatever he likes. He can sell his completed projects to other entrepreneurs and start new ones, expand his existing projects, or hire a management team, sit back, and enjoy the fruit of his past labor. No matter what each decides, however, the entrepreneur must keep in mind that markets are in a continuous state of flux, and that there are other entrepreneurs standing in the wings eager to gain entry. Insofar as profit maximization is a means to guard against market competition, it is probably not unreasonable to assume that all entrepreneurs are -- at least to a large extent -- profit maximizers. With a model of unitary output, however, we are unable to capture the individual behavior of sole entrepreneurs and are compelled to treat all of entrepreneurial activity as that of a single entrepreneur.

Our first goal in the modeling of entrepreneurial behavior must be the incorporation of our value-added cost function into the entrepreneur's decision function; else we abandon what we set out to show. After careful investigation and much experimentation the following unconstrained optimization problem was selected.

$$\max_{\text{w.r.t. } t^p \text{ and } I} f(t^p, \Phi)K(t^p, I) - (1 + r^e)I \quad (\text{equation 32})$$

Taken together the expressions  $f(t^p, \Phi)$  and  $K(t^p, I)$  represent the expected future value of an investment made at the beginning of the current production cycle ( $t_0$ ). The expression  $(1 + r^e)I$  represents the value that must be returned to investors at the current cycle's end ( $t_1$ ). The goal of the entrepreneur is to maximize this difference at  $t_0$ . Let us consider first the expression for the expected future value of new investment ( $I$ ). Though it likely appears odd at first glance that the capital cost function is included on the revenue side of equation 32, we have only to recall that one, capital formation is the business of the entrepreneur; and two, at the right price, the entrepreneur is able to sell whatever and all that he produces. In effect, this model assumes that the entrepreneur has perfect knowledge with regard to the kinds of goods and services consumers want, and that consumers will always want more of the goods and services that he produces. Obviously, a very enviable market to work in, but well suited to our purpose.

Except for the introduction of the projected time table ( $t^p$ ) and level of technology ( $\Phi$ ) all variables are as before. The projected time table, in particular, is the expected duration of the current production cycle -- namely, the time believed necessary to implement new investment over and above that necessary to replace worn capital and maintain liquidity in production. It replaces the value  $t$  in the capital cost function given by equations 4 and 8. In effect, rather than looking backward at what just occurred between  $t_1$  and  $t_0$ , we are now looking forward in anticipation of what is to happen between  $t_0$  and  $t_1$ . This said, we have not entirely abandoned the past in our forward outlook. Although the value of  $K$  changes, the function itself remains the same. Also new is the introduction of the investment function  $f(t^p, \Phi)$  that we will discuss in greater detail very soon.

## THE ENTREPRENEURIAL INVESTMENT AND CAPITAL COST FUNCTIONS

When the entrepreneur decides how much new capital to raise he must consider the performance of his past projects -- namely,  $r_0$ , as well as the potential performance of future projects (equation (32)). On the one hand, new technology makes old technology obsolete; and on the other hand, investment shortfall or overinvestment create imbalances in the entrepreneur's anticipated real rate of return ( $r_1$ ).<sup>28</sup> In consequence, the expression  $f(t^p, \Phi)K(t^p, I)$  constitutes the entrepreneur's expectations with regard to the future value of his undertaking based on his past performance. Let us examine the capital cost function more closely in order to understand why this is so. In so doing let us recall equation 8.

$$K(t, I_{-1}) = e^{rt} \quad (\text{equation 8})$$

The value of  $r$  in equation 8 is derived from a mathematical approximation of observed market phenomenon. Its value is determined by a decision made with regard to  $t^p$  and  $I$  at the beginning of the previous production cycle just ended. As  $t_0$  marks both the end of the previous production cycle and the beginning of the current production, what was an unknowable anticipated outcome ( $t_{-1}^p$ ) at the beginning of the previous cycle is now a realized observed known ( $t_0$ ) at the beginning of the current cycle. In contrast, the value of  $I_{-1}$  is both decided and observed at the beginning of the previous cycle. As such,  $t_0$ ,  $I_{-1}$ , and  $r$  are all known when the entrepreneur makes his decision to invest at the beginning of the current production cycle. In effect,  $r$  becomes an implicit parametric given in equation 32 that serves as an implicit constraint on the entrepreneurial investment decision of the current cycle. Accordingly, although we assume that entrepreneurs can, on average, properly guess  $t^p$ , no entrepreneur can ever know with certainty how long it will take to complete an undertaking before it is actually completed. As a result, where  $t_0^p$  is observable at the beginning of the current production cycle,  $t_1$  is not.

In contrast with the cost of capital function, the entrepreneurial investment function  $f(t^p, \Phi)$  has only one decision variable -- namely,  $t^p$ . Mathematically, the value of the function is best conceived as an index whose value fluctuates around unity. Conceptually, it is a measure of the entrepreneur's eagerness or *propensity* to invest given some level of technology and the entrepreneur's estimated timetable for completion of his new project. The value of  $f(t^p, \Phi)$  is set to one when  $t^p$  equals  $(t_0 - t_{-1})$ , and  $\Phi$  is the same as that of the previous production cycle just-ended.

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28 Whereas investment shortfall means a potential deterioration of the current capital base; overinvestment means an inability to satisfy investor demands without incurring personal losses. As a result, the entrepreneur's decision to undertake new projects or maintain old ones depends in part on the absence or presence of imbalances in the previous production cycle. For the moment, we will disregard these temporal imbalances and assume that, when taken together, entrepreneurs accurately predict the length of implementation of their intended projects. In other words, where one entrepreneur overestimates, the other underestimates, but on balance they are both able to meet their projected due dates. This is not an unreasonable assumption, by the way, for entrepreneurs who cannot properly forecast the length of their undertakings are likely to be driven from the market place. Although we assume that entrepreneurs are generally good at predicting the length of their projects, this is obviously not always the case. The best example of this is the ever reoccurring boom-bust cycles of modern times, when large numbers of entrepreneurs fail simultaneously in their ability to select proper values for  $t^p$  and  $I$ . Indeed, the purpose of this paper is to provide a framework for understanding this *en masse* miscalculation.

### The Model's Components

In general, we assert  $\delta f / \delta t^p < 0$  (equation 33)  
and  $\delta f / \delta \Phi > 0$  (equation 34)

The assertion expressed in equation 33 is probably obvious insofar as the introduction of new technology is what spurs entrepreneurs to seek additional funds from among investors. It is also what given them the gumption to risk their career and reputation should they fail to provide their promised return. The assertion provided in equation 34 also appears reasonable insofar as the longer the projected time frame for implementation, the greater the risk of the project's incompleteness and the greater the cost of new capital formation. Thus, unless the potential return of large projects that are difficult to implement is significantly higher than that of smaller projects whose time to completion is generally short, smaller projects will be preferred.

The relationships expressed by the formulation  $f(t_p, \Phi)K(t_p, I)$  assert clearly that entrepreneurial decision making, unlike that of the animalistic spirits proposed by Keynes, is driven by technological innovation and subject to market constraints. Of particular interest is the inequality expressed in equation 33, for where  $\delta f / \delta t^p$  is negative,  $\delta K / \delta t^p$  is positive. Although this relationship introduces uncertainty into the model it also serves as a well-founded check on the entrepreneur's willingness to seek new investment funds. The variable  $\Phi$  is excluded from the capital cost function, because its effect on  $K$  is knowable, but generally undetermined. Although the implementation of new technology will always require an increase in capital expenditure at the beginning of a production cycle, its overall effect on capital formation is uncertain. Whereas some technology reduces the cost of production, other technology only improves the quality of the output.

In summary, then, the entrepreneurial imperative is to produce more or better with the smallest investment in the shortest time frame possible.

Combining the entrepreneurial investment and capital cost functions yields the expected future value of new investment estimated at the beginning of the current production cycle. In effect,

$$f(t^p, \Phi)K(t^p, I) = f(t^p, \Phi)e^{rt} \quad (\text{equation 35})$$

where  $r$  and  $\Phi$  are both givens, and the  $t$  of expression  $e^{rt}$  is, in fact,  $t^p$ .<sup>29</sup> This relationship highlights the constraining nature of  $r$  as a parameter from the previous production cycle. Notice further that when the value of  $f$  is unity, the above relation reduces to equation 8 projected forward.

### THE COST OF NEW INVESTMENT

We will now turn to the expression  $(1 + r^e)I$  of equation 32. The value  $r^e$  is the expected real rate of return on new investment; it is the value to which entrepreneurs and investors (see equations 30 and 31a above) agree when they negotiate the price for, and the quantity of new investment for the current period in the

<sup>29</sup> My software does not permit greater numerical precision.



## The Model's Components

market for new investment (figure 5). In effect,  $I$  is the agreed amount invested at the beginning of the current production cycle, and  $I + r^e I$  is what must be returned at the cycle's end. Although one may be inclined to view this return as the principal and interest on a loan, it is better to view it as the value of finished goods set aside at the beginning of the production cycle and the value of finished goods and services made available for purchase at its end.

As stated earlier there is no compelling reason in this model for new investment to take the form of a bank loan. Further, when an investor surrenders money to an entrepreneur, the investor cannot spend it on goods and services that he might otherwise wish to purchase. In effect, he places the purchasing power over those finished goods and services in the hands of the entrepreneur who, in turn, surrenders this power to factor owners in exchange for their inputs during the course of the production cycle.

### SOLVING FOR THE INVESTMENT DEMAND FUNCTION

In order to solve for the demand function of new (and maintenance of old) investment we must first find the solution to the maximization problem given in equation 32. Differentiating with respect to  $t^p$  and  $I$  obtains the following two first order conditions necessary to insure a maximum:

$$f_t K + f_{K_t} = 0 \quad (\text{equation 36a})$$

$$f_{K_t} - (1 + r^e) = 0 \quad (\text{equation 36b})$$

The sufficient second order conditions are given as follows:

$$\delta^2 \pi / \delta t^{p2} = f_{tt} K + 2f_{tK_t} + f_{K_{tt}} < 0 \quad (\text{equation 37a})$$

$$\delta^2 \pi / \delta I^2 = f_{K_{tt}} < 0 \quad (\text{equation 37b})$$

$$\text{and} \quad (f_{tt} K + 2f_{tK_t} + f_{K_{tt}})f_{K_{tt}} - (f_{tK_t} + f_{K_{tt}})^2 > 0 \quad (\text{equation 37c})$$

where  $\delta^2 \pi / \delta t^p \delta I = \delta^2 \pi / \delta I \delta t^p$  and  $f_t K_t + f_{K_{tt}} = f_t K_{tt} + f_t K_{tt}$

Before solving for our investment demand function it may prove useful to discuss in some detail the nature of our first order conditions. Rearranging equation 36a and dividing through by  $K$  yields the following relationship

$$f_t = -f(K_t/K) \quad (\text{equation 38})$$

Noting that  $K_t/K = r$  obtains

$$f_t = -fr \quad (\text{equation 39})$$

Setting  $f$  equal to 1 and substituting into equations 39 and 36b and then rearranging 36b yields

$$f_t = r \quad (\text{equation 40a})$$

$$\text{and} \quad K_t = r^e \quad (\text{equation 40b})$$

These are conditions for long-term general equilibrium in the absence of a change in the level of tech-

### The Model's Components

nology ( $\Phi$ ) and the time-table for new investment ( $t^p$ ). As we can easily see, under these conditions the expected real rate of return ( $r^e$ ) decided in the investment market must equal the change in the cost of capitalization per unit change in new investment.

Solving for  $I^* = I(\Phi, r^e)$  and  $t^{p*} = t^p(\Phi, r^e)$  in equations 36a and 36b yields the following demand function for new investment

$$I^D = I(\Phi, r^e) \quad (\text{equation 41})$$

Substituting back into our first order conditions with  $I^*$  and  $t^{p*}$  yields the following pair of identities from which we can solve for the relevant comparative statics.

$$f_t(t^{p*}, \Phi)K(t^{p*}, I^*) + f(t^{p*}, \Phi)K_t(t^{p*}, I^*) \equiv 0 \quad (\text{equation 42a})$$

$$f(t^{p*}, \Phi)K_t(t^{p*}, I^*) - (1 + r^e) \equiv 0 \quad (\text{equation 42b})$$

Differentiating with respect to  $r^e$  and solving for  $\delta t^{p*}/\delta r^e$  and  $\delta I^*/\delta r^e$  produces the following pair of results

$$\delta t^{p*}/\delta r^e = -(f_t K_t + f K_{tt})/D < 0 \quad (\text{equation 43a})$$

$$\delta I^*/\delta r^e = f_{tt} K + 2f_t K_t + f K_{tt} < 0 \quad (\text{equation 43b})$$

where  $D > 0$  is the value of the determinant obtained from the Hessian matrix that forms the coefficients for  $\delta t^p/\delta r^e$  and  $\delta I^*/\delta r^e$  in the system of equations that results. It should be noted that  $K_{tt} = \delta V/\delta I$  which we know to be greater than zero from equations 1 and 2b. Further, we know  $f_{tt} K + 2f_t K_t + f K_{tt} < 0$  from our second order condition given in equation 37a.

Rewriting equation 43b as  $\delta I^D/\delta r^e < 0$  (equation 44)

provides us with the slope of the investment demand curve (see figure 5). As we cannot know the effect of a change in  $\Phi$  on  $K$ , it is impossible to know without further assumption the signs of  $\delta t^{p*}/\delta \Phi$  and  $\delta I^*/\delta \Phi$ . Substituting back into the objective function with  $t^{p*}$  and  $I^*$  and applying the envelope theorem, we are able, however, to make the following two determinations:

$$\delta \pi^*/\delta \Phi = \delta \pi/\delta \Phi = f_{\Phi}^* K^* > 0 \quad (\text{equation 45a})$$

$$\delta \pi^*/\delta r^e = \delta \pi/\delta r^e = -I^* < 0 \quad (\text{equation 45b})$$

In other words, in the absence of a change in the expected real rate of return ( $r^e$ ) an improvement in the level of technology leads to increased profit -- this only, however, if entrepreneurs make the proper decisions with regard to the amount of new investment ( $I^*$ ) and the time-table ( $t^{p*}$ ) for their projects, and if they are able to realize them.

A final note before moving on to the market for new investment. Differentiating equation 8 with respect to  $t$  and  $I$  for the current period yields the following relationship:

## The Model's Components

$$K_t \delta t + K_l \delta l = r^{ert} \delta t \quad (\text{equation 46a})$$

Rearranging obtains  $\delta l / \delta t = (rK - K_l) / K_l \leq 0$  (equation 46b)

As all of the terms to the right of the equal sign in equation 48b are positive, it is impossible to know the sign of  $\delta l / \delta t$  without more information.

### THE MARKET FOR NEW INVESTMENT

In figure 5 the investment demand and supply functions (equations 30 and 41, respectively) are wedged together to form the market for new investment (or, more simply, the investment market).<sup>30</sup> All of the information required to understand the graphical mechanics of this market is contained in equations 31a, 31b and 31c, and in equation 44.

Equating the supply of new investment (equation 30) and the demand for new investment (equation 41) and solving for  $r^e$  gives us the equilibrium, market-clearing price ( $r^{e*}$ ) -- namely, the negotiated expected real rate of return (equation 48a) for the new (current) production cycle. Based on this negotiated rate, a level of new investment is decided (equation 48b), investors relinquish their claim to the agreed amount, and entrepreneurs begin work on the realization and/or renewal of their new and previous projects, respectively.<sup>31</sup>

$$I^S = I^D \quad (\text{equation 47a})$$

$$I(r^e; S) = I(r^e; \Phi) \quad (\text{equation 47b})$$

$$r^{e*} = r^e(S, \Phi) \quad (\text{equation 48a})$$

Substituting with  $r^{e*}$  into either the demand or supply functions yields the equilibrium quantity of new investment -- namely,

$$I^* = I(S, \Phi) \quad (\text{equation 48b})$$

As both  $Q$  and  $r$  are treated as parameters of the current production cycle -- i.e., unalterable givens, foregone events of the previous production cycle --, we need only be concerned about the values of  $S$  and

30 This market is more commonly known as the *loanable funds market* and is the very market that Keynes rejected in his General Theory! In his errant effort to describe the *real world*, as opposed to a *real economy* that had been usurped by the introduction of debt-based currency and fiat money, Keynes replaced the loanable funds market with a market for money and proceeded to legitimize, thereby, the counterfeiting of money by the state and private banks under the guise of scientific investigation.

31 Once again, depending on the nature of the agreement, the transfer of funds from the investor to the entrepreneur, could take a variety of forms. It could be a loan with interest that must be paid back at the end of the production cycle; it could be a stock purchase with a fixed dividend and an expected resale price not less than the amount of the original purchase. It could be the sale of a bond with a fixed rate of interest and an expected resale value equal in worth to that of the original purchase price. For the model itself, all that is important is that investors relinquishes their right to the finished goods and services for which the entrepreneur assumes command, and that this right can be transferred to the owners in compensation for factor inputs later utilized by the entrepreneur.

## General Equilibrium

$\Phi$ . Insofar as the sign of  $\delta I^*/\delta \Phi$  is indefinite, and  $S$  and  $\Phi$  appear on opposite sides of the market, we will focus our attention on  $S$ .

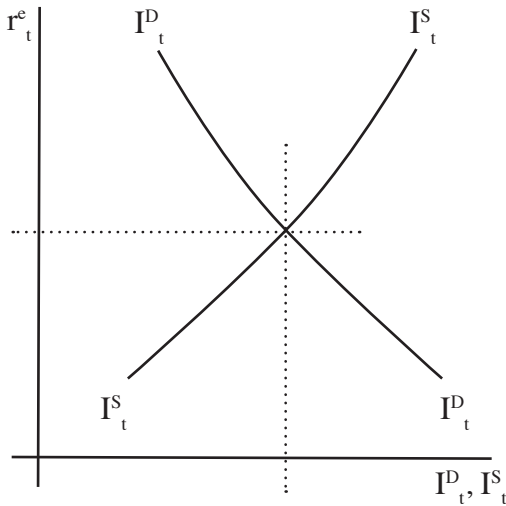


Figure 5. The Market for New Investment

Before advancing the discussion further, however, let us pause briefly and review our understanding thus far. Firstly, the value of  $t$  in the expression  $Q = V(t_0, I_{-1})$  (see equation 18c) represents both the starting point of the new (or current) production cycle and the terminal point of the previous production cycle just-ended. In effect,  $t$  determines the value of  $V$  for the production cycle just ended and therefore  $Q$  for the production cycle just beginning.  $Q$  becomes then a given for the beginning of the new production cycle. Secondly, the value of  $t$  in the expression  $K(t, I)$  (see equation 4) represents the length of the interval over which  $K$  has been evaluated. Accordingly, it is a measure of duration and equal in value to  $(t_0 - t_{-1})$  for the just completed production cycle. For the current production cycle, however,  $t^p$  is the result of an informed decision on the part of entrepreneurs; it is not necessarily observed, but assumed to be known by the entrepreneur when he makes his decision about how much to invest. Thirdly, the value  $r$ , though

observed for the production cycle just ended and treated as a given for the production cycle just beginning, may change by the end of the current cycle just beginning when it is recalculated and fixed again for the production cycle subsequent to the current cycle. Fourthly, whereas  $t_0 - t_{-1}$  always refers to the duration of the cycle just concluded,  $t^p$  always refers to the expected duration of the cycle just beginning. Finally, whereas the level of  $Q$  in the investment demand function is backward looking, the levels of  $C^*$  and  $I^*$  determined by the utility maximization problem are forward-looking insofar as they are based on the expected real rate of return ( $r^e$ ) negotiated at the beginning of the new (or current) period. Similarly, while the real rate of return ( $r$ ) used to define the capital cost function  $K(t^p, I^*)$  of the current period is backward-looking, the level of new investment ( $I^*$ ) and the projected time-frame ( $t^{p*}$ ) are both forward-looking.

We are now in a position to assemble the components of our general equilibrium model and discover its usefulness.

## General Equilibrium Modified Garrison Triad

As stated at the outset of this paper, the inspiration for this mathematical model came from the conceptual model created by Roger Garrison in his work *Time and Money*.<sup>32</sup> Though graphically similar in overall design, this model is, however, conceptually very different from Garrison's original triad in several regards. Indeed, only the investment market depicted in the 4th quadrant of figure 6 is very close. The other

<sup>32</sup> Garrison (2001), The original form of this three component model appears to be the unique creation of Roger W. Garrison. I refer to it in this paper as Garrison's triad.

similarities including the trade-off between current and future consumption, the time-value of capital, the real and expected real rates of return and their inter-temporal coordination are all approached very differently.

In quadrants one, two, and four of figure 6 are depicted figures 4, 3, and 5, respectively. The seeming lack of ordinal correspondence disappears upon inspection of the figure in which everything falls rigorously into place. Initially, the graphical mechanics and comparative statics of the model are best understood from a point of zero-growth, general equilibrium.

### ZERO-GROWTH EQUILIBRIUM

Zero-growth general equilibrium implies that neither the level of technology ( $\Phi$ ), nor the expected real rate of return ( $r^e$ ) change across time. For in the absence of supply shocks only changes in technology and the expected real rate of return can bring about a change in the entrepreneur's decision to seek investment for the purpose of implementing new projects. Further, in the absence of technological advancement only a change in the level of savings ( $S$ ) can bring about a change in the expected real rate of return. Thus, our two conditions for zero-growth equilibrium are that both  $\Phi$  and  $S$  remain constant. However, these conditions are only sufficient insofar as entrepreneurs solve their maximization problem correctly in the previous period and potential supply shocks are ignored.

### A CHANGE IN THE LEVEL OF SAVINGS

With no change in the level of technology ( $\delta\Phi = 0$ ) the effect of a change in  $S$  on  $I^*$  is certain (see equation 31b). So, let us assume that consumer-investors, when taken together, decide one day to decrease their savings.<sup>33</sup>

At the current level of utility consumer-investors would prefer to invest more and consume less (see dotted budget constraint in quadrant 1 of figure 7), and the supply curve for new investment shifts outward (see equation 31b).<sup>34</sup> This results in downward pressure on the expected real rate of return ( $r^e \downarrow$ ) on the one hand (see 4th quadrant, figure 7), and upward price pressure on  $P_3$  on the other (see 1st quadrant). The fall in the expected real rate of return, encourages entrepreneurs to undertake new investment projects that they would have previously ignored ( $I^* \uparrow$ ). Although the model suggests a corresponding fall in consump-

<sup>33</sup> Although hoarding, usually brought about by the fear of a future shortage of goods and services, is the more popular scenario, the opposite has been chosen. The reason for this choice will become clear in the paper subsequent to this one.

<sup>34</sup> The assumption here is that consumer-investors generally prefer an increase in growth over one of decline. Indeed, the budget constraint now intersects the utility level curve in two places: one that would result in a fall in new investment, and one that results in an increase in new investment. See Hoppe, 2001.

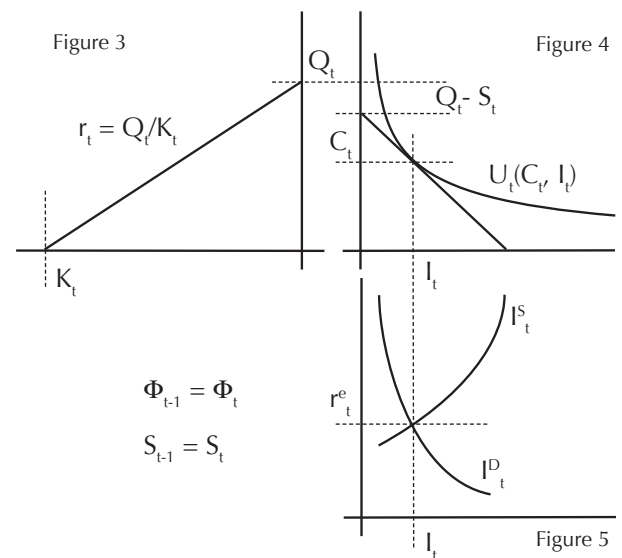


Figure 6. Zero-Growth Equilibrium



tion, this need not be the case, as the release of savings has left factor owners -- at least in the short-term -- with more of everything. Thus, we can assume that utility begins to rise (not depicted).

Although the effect on  $I^*$  is clear, the effect on  $t^{p*}$ , as we have already shown, is not. This said, a little economic intuition is useful. In order to implement their new undertakings entrepreneurs must either draw already employed factor inputs away from their current uses, or gather additional factor inputs that have not been employed. Either scenario requires additional time, thus we can assume that  $t^p$  rises. This rise in  $t^p$  has a dampening effect on entrepreneurial profits and the value of  $f$  drops below unity. Important in this scenario is that the increase in new capital investment is affordable, as there are now more real goods in the system to pay factor owners for their additional input.

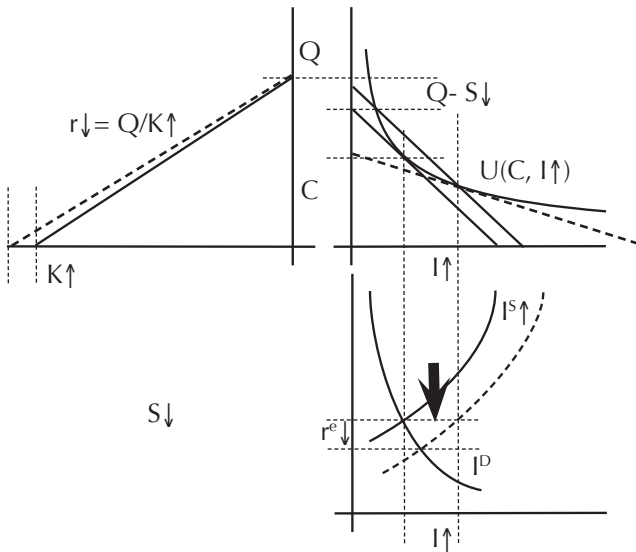


Figure 7. A Fall in Savings

Although the model constrains the behavior of all agents (factor owners, consumer-investors, and entrepreneurs) to behave in a certain way at the beginning and the end of each production cycle, what happens during the production cycle is not well-described. Once again, we must invoke economic intuition.

Initially, the formation of new capital above that required to replace worn capital and facilitate the use of old requires the cost of capital to rise ( $K \uparrow$ ). This rise is indicated by the outward rotation of the hypotenuse of the triangle depicted in quadrant 2, figure 7. As the newly formed capital can have no effect on total output until the end of the production cycle, there is a short-term fall in the real rate of return ( $r \downarrow$ ). This decrease corresponds with the observed decrease in the expected real rate of return ( $r^e \downarrow$ ) in the investment market. This outward rotation is unlikely to take place all at once, as new capital formation is a gradual process, and its cost one of steady accumulation.

By the time we reach the end of the production cycle, this gradually accumulating cost is realized as finished output and there is a sudden rise in  $Q \uparrow$ . As a result, the real rate of return rises to its previous level ( $r \uparrow$ ). These rises are reflected in quadrant 2, figure 8 by the further outward rotation of the triangle's hypotenuse, but this time from the opposite pivot point. The rise in output ( $Q \uparrow$ ) also causes the budget constraint in quadrant 1 to rise. This rise is offset in part by a rise in savings ( $S \uparrow$ ), as factor owners return to their previous savings habits. Since we cannot know with the information given which is greater -- the change in savings or the change in output -- we will assume that they are equal for graphical simplicity and leave the budget constraint rest in its current position. Where the new higher level of investment comes to rest is at the point where the first order conditions of utility maximization are achieved. In order for equilibrium to be achieved, however, the expected real rate of return ( $r^e$ ) must return to its previous

## Conclusion

level. This can only occur with a simultaneous leftward shift in the supply of new investment and a rightward shift in its demand. The leftward shift can be explained by the decrease in the relative price of new investment to current consumption. The increase in finished goods and services available for current consumption at the end of the production cycle, relieves the upward price pressure on new investment that was set into motion by the decrease in savings at the beginning of the cycle. The rightward shift in the demand for new investment on the part of entrepreneurs can be explained by the need for new replacement capital given the higher level of capital formation that was achieved during the production cycle just ended. In addition, once the new capital formation has been put into place the length of time required to employ this new capital is shorter than what it was when the new capital was first introduced, and the entrepreneur's propensity to invest returns to unity.

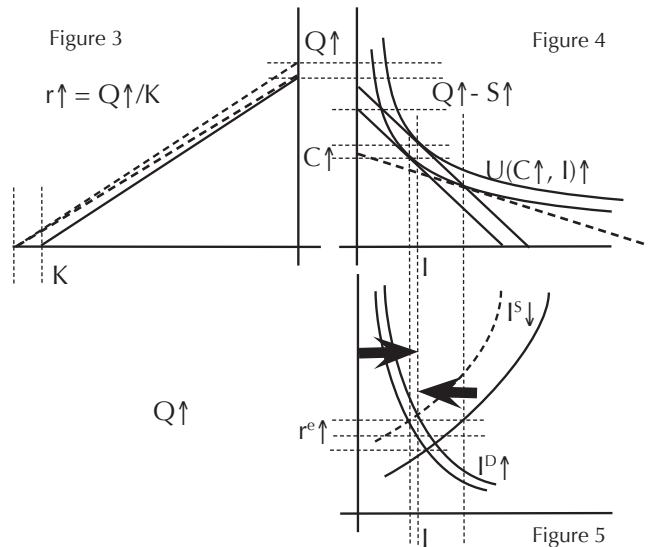


Figure 8. A Rise in Real Output

## Conclusion

### Closing the Door on Keynesian Folly

#### THEORETICAL CRITIQUE AND COMPARISON

Surely, the shortcomings of this model require little elaboration. The absence of a direct mathematical link between the current and concurrent production cycles and a similar absence to account for changes in the real rate of return from one production cycle to the next are all too apparent. Then too, one can only wonder why many an economist would even care. After all, Keynesian economic theory has long prospered with only disjunct connections between the short-term effects of government and central bank intervention and their long-term consequences on economic growth and stability.

As undergraduate students of economics we are taught to treat banks as endogenous players subject to both a buyers- and a sellers-market. Entrepreneurs on the other hand are completely ignored, or they are treated as exogenous players who enter and leave the market place at their own discretion and according to whim. This model shows clearly what happens when entrepreneurs are treated as actively participating market agents who purchase capital and factor inputs and sell output. Indeed, we have completely and purposefully eliminated bankers as an original source of funding. For, it is with this omission that we are able to demonstrate (see follow-up paper) the pernicious effects of modern banking on a real economy. This said, we already have enough to expose Keynesian economists for the charlatans that they truly are.

## Conclusion

### KEYNESIAN FOLLY AND THE RETURN TO MACROECONOMIC SANITY

A typical, undergraduate Keynesian approach to our world's macroeconomies divides each into two major sub-markets and two sets of economic agents. On the one side, are consumers who sell their labor services in exchange for income in a national labor market, and who use this income to purchase finished goods and services in a national product market. On the other side, are producers who purchase the labor services of consumers in the previously mentioned national labor market and sell their finished goods and services in the aforementioned national product market. A third additional, national market is then introduced in which consumers lend money to producers in the form of savings who borrow these in the form of private sector investment via financial intermediaries called banks. Finally a government agent is introduced who, we are told, is not really an economic agent -- this, despite its relentless intervention in market activity. Ignoring the rest of the world's economies and focusing solely on the domestic market place of each this four-agent, three-market scenario is then summarized with two national income accounting identities and an empirically derived consumption function similar to the following:

$$Y \equiv C + I + G \quad (\text{equation 49})$$

$$Q \equiv C + S + T \quad (\text{equation 50})$$

$$C = a + bY \quad (\text{equation 51})$$

From these three equalities is constructed the famed Keynesian cross (figure 9) that Keynes himself probably never drew, but that every neophyte student of macroeconomics memorizes assiduously and regurgitates *ad nauseum* on his undergraduate examinations. For it is from this simple and rather innocuous looking diagram that we are taught about the investment and government spending multipliers and how small changes in investment (I) and government spending (G) can grow and shrink our national economies many fold. From this same diagram we are taught that being thrifty does not lead to greater savings and more investment, but to lower income and economic impoverishment (the paradox of thrift). Somewhat later in our undergraduate economic studies we are told about the market mechanisms (IS-LM analysis) by which the balance between investment and savings is maintained and how a central bank or government can manipulate interest rates, private sector investment, and national output through simple changes in our nation's money supply. This magic depends, of course, on several very important assumptions including the stickiness of product and wage prices in the short-run, monopoly control of the nation's money supply in the long-run, and the incredible belief that the long term health of a national economy can be best managed by fixing the price of labor and investment from a national central planning agency in all runs.

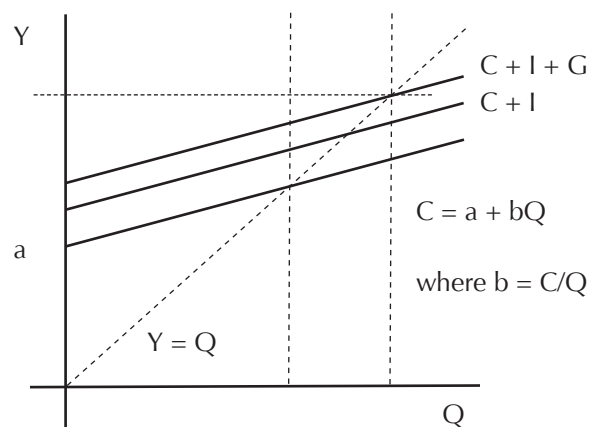


Figure 9. Keynesian Cross

No matter, let us play along and see how things turn out in the model developed in this paper.

## Conclusion

Solving for  $Y$  in equation 51, substituting the result into equation 54, setting  $Y$  equal to  $Q$ , and rearranging terms obtains:<sup>35</sup>

$$C = (a + bG)/(1-b) + bI/(1-b) \quad (\text{equation 52})$$

Let us now graph equation 52 against the utility maximization problem of figure 4 of this paper (see figure 10). The two sets of double-headed arrows show the direction of increasing and decreasing consumption and investment in the magical wonderland of Keynesian macroeconomics and their respective movements in the commonsensical, real world of classical microeconomic trade-offs.

Rest assured, markets do not work properly in the magical wonderland of Keynesian economists, for there is no trade-off between current and future real consumption. Indeed, in this magical wonderland one simply advances to higher and higher levels of national utility by increasing the money supply. In short, Keynesian economists would have us believe that you can defy the law of economic gravity that nothin' comes from nothin'. It is a proposition that even a well-educated middle-schooler would resist, but is easily swallowed by 100s of millions of undergraduate students world-wide, because it is taught from master magicians with PhD's.

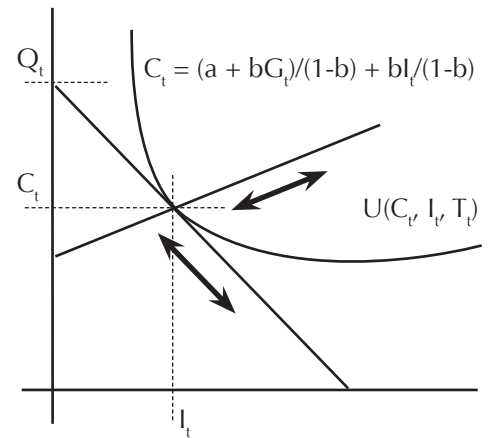


Figure 10. The Keynesian Cross Revisited

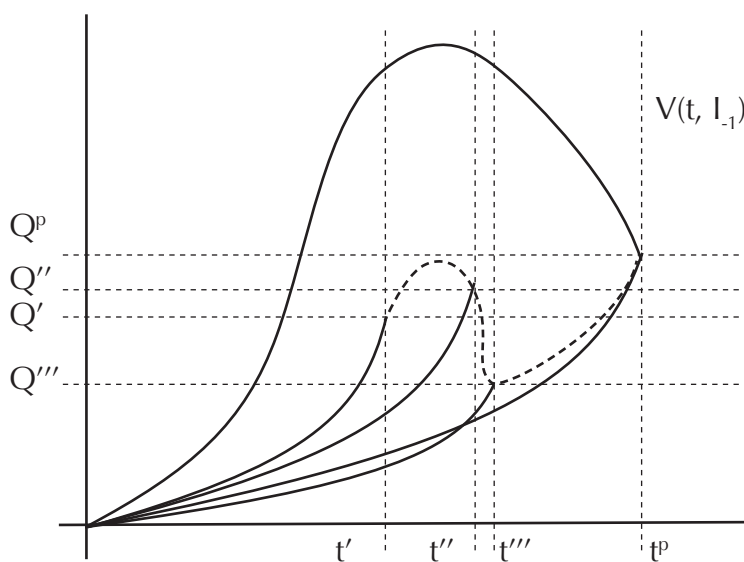
<sup>35</sup> Garrison (2001), p. 136.

## Appendix

Early in the paper it was pointed out that the shape of the value-added cost function could vary from production cycle to production cycle, and that the selected contour was fairly arbitrary in nature. Insofar as the function was used merely as a tool to derive Hayek's triangle and the subsequent mathematical formulation of the real rate of return ( $r$ ), the somewhat arbitrary nature of the functional form is completely permissible. Having thus established the basic framework of the model we can now entertain a more realistic discussion of the actual shape of the value-added cost function.

So long as we are modeling the production of a unitary final good and service, the above discussion is completely satisfactory. Indeed, to the extent that we focus our attention solely on the current and preceding production cycles the above formulation is largely sufficient. Insofar as the ability of factor owners to realize just remuneration for their inputs during the course of the current production cycle depends on the realized output of concurrent production cycles, the actual shape of each concurrent production cycle can be of crucial importance. Indeed, in the above discussion we have assumed that the value-added cost function of all production cycles are similar in shape.

In keeping with the notion that  $t^p$  represents the projected timetable of the longest production process we can well imagine shorter production processes that begin at the same  $t$ , but whose final output is realized much sooner than that of the timetable of the longest investment project. Under such a scenario the shape of the value-added cost function for each production cycle would likely appear similarly to that depicted in figure A1, where  $t'$ ,  $t''$ , and  $t'''$  represent the projected time-tables of shorter production processes, and  $Q'$ ,  $Q''$ , and  $Q'''$  the added-cost value of their realized final output at each corresponding  $t$ . Although we could still calculate  $K$  just as before, the handling of the value of total output for the entire production cycle would be more difficult without significant manipulation of the time of sale of the completed output or a corresponding adjustment of the time-value of revenue received from the its sale. Accordingly, we would no longer be dealing with a unitary output and the valuation of total output would become more complex. As a result, the calculation of the real rate of return ( $r$ ) would no longer be as straight forward as before. These complications are mentioned in passing, for their consequence in the formation of inflationary bubbles as treated in the subsequent paper are of some importance, and it is worthwhile to allow them to linger in the background as the model is further developed.





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